Long-range correlated random field and random anisotropy *O(N)* models: A functional renormalization group study

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We study the long-distance behavior of the $O(N)$ model in the presence of random fields and random anisotropies correlated as $\sim 1/x^{d-\sigma}$ for large separation x using the functional renormalization group. We compute the fixed points and analyze their regions of stability within a double $\varepsilon = d - 4$ and σ expansion. We find that the long-range disorder correlator remains analytic but generates short-range disorder whose correlator develops the usual cusp. This allows us to obtain the phase diagrams in (d, σ, N) parameter space and compute the critical exponents to first order in ε and σ. We show that the standard renormalization group methods with a finite number of couplings used in previous studies of systems with long-range correlated random fields fail to capture all critical properties. We argue that our results may be relevant to the behavior of ^{3}He-A in aerogel.

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I. INTRODUCTION

The effect of weak quenched disorder on large-scale properties and phase diagrams of many-body systems attracted considerable attention for decades. Despite significant efforts, there still remain many open questions. The prominent example is the $O(N)$ model in which an N-component order parameter (the magnetization in the spin system notation) is coupled to a symmetry-breaking random field. For $N = 1$, it is known as the random field Ising model (RFIM).[1] For $N > 1$, one has to distinguish the random field (RF) case where the order parameter couples linearly to disorder, and the random anisotropy (RA) case, where the coupling to disorder is bilinear. These models are relevant for a variety of physical systems such as amorphous magnets,[2] diluted antiferromagnets in a uniform external magnetic field,[3] liquid crystals in porous media,[4] critical fluids in aerogels,[5] nematic elastomers,[6] and vortex phases of impure superconductors.[7] For $N = 1$, the RA model reduces to the random-temperature model, where the randomness couples to the local energy density as, for example, in diluted ferromagnets.[8] In contrast to the systems with random-temperature-like disorder, the RF and RA models suffer from the so-called dimensional reduction (DR). A straightforward analysis of the Feynman diagrams giving the leading singularities yields to all orders that the critical behavior of the RF $O(N)$ model in d dimension is the same as that of the pure system in $d - 2$ dimensions.[9] Consequently, the lower critical dimension is $d_{\rm lc}^{\rm DR}(N = 1) = 3$ for Ising-like systems and $d_{\rm lc}^{\rm DR}(N > 1) = 4$ for systems with continuous symmetry. This can elegantly be demonstrated using supersymmetry.[10] However, simple Imry-Ma arguments show that the lower critical dimension of the RFIM is $d_{\rm lc}(N = 1) = 2$.[11] The deviation from DR is also confirmed by the high-temperature expansion.[12] Thus DR breaks down, rendering standard field theoretic methods useless.

Another known problem where the perturbation theory is spoiled by DR is elastic manifolds in disordered media. There, two methods were developed to overcome difficulties related to DR: the Gaussian variational approximation (GVA) in replica space and the functional renormalization group (FRG). The GVA is supposed to be exact in the limit $N \to \infty$. Unfortunately, this approach when applied to the RF problem leads to very complicated equations which do not allow us to compute the critical exponents.[13] Considering the RF $O(N)$ model, Fisher[14] showed that expansion around the lower critical dimension $d_{\rm lc} = 4$ generates an infinite number of relevant operators which can be parametrized by a single function. However, he found that the corresponding one-loop FRG equation has no analytic fixed point (FP) solution. Only recently, using the progress in the elastic manifold problem,[15,16,17,18] it was realized that the scaling properties of systems exhibiting metastability are encoded in a nonanalytic FP. Feldman[19] has shown that, indeed, in $d = 4 + \varepsilon$ and for $N \geq N_{\rm c} \approx 3$ (a more precise computation[20] gives $N_{\rm c} = 2.834\,74$), there is a nonanalytic FP with a cusp at the origin. This FP provides the description for the ferromagnetic-paramagnetic phase transition in the RF $O(N)$ model and allows one to compute the critical exponents which are different from the DR prediction. Recently, Le Doussal and Wiese[20] extended the FRG analysis to two-loop order. The extension beyond one-loop order is highly nontrivial due to the nonanalytic character of the renormalized effective action, which leads to anomalous terms in the FRG equation.[18] The two-loop calculations were also independently performed in Ref. 21, and the truncated exact FRG was proposed in Ref. 22. The more accurate analysis of the FRG flows for the RF model showed that for $N > N^* = 18 + O(\varepsilon)$ there is a crossover to a FP with weaker nonanalyticity resulting in the DR critical exponents.[20,21,22,23] A similar picture was found for the RA $O(N)$ model with the main difference that $N_{\rm c} = 9.4412$ and $N^* = \infty$.[20]

A more peculiar issue concerns the phase diagram of the RF and RA models below $d_{\rm lc}$. It is known that for the RF model and models with isotropic distributions of random anisotropies true long-range order is forbidden below $d_{\rm lc} = 4$ (for anisotropic distributions, long-range order can occur even below $d_{\rm lc}$).[24] However, quasi-long-range order (QLRO) with zero order parameter and an infinite correlation length can persist even for $d < d_{\rm lc}$. For instance, the GVA predicts that

the vortex lattice in disordered type-II superconductors can form the so-called Bragg glass exhibiting slow logarithmic growth of displacements.[25] This system can be mapped onto the RF $O(2)$ model, in which the Bragg glass corresponds to the QLRO phase. Indeed, for $N < N_c$ and $d < d_{\rm lc}$, the FRG equations have attractive FPs which describe the QLRO phases of RF and RA models.[26] In order to study the transition between the QLRO phase and the disordered phase expected in the limit of strong disorder, one has to go beyond the one-loop approximation. The truncated exact FRG (Ref. 22) and the two-loop FRG (Ref. 20) performed using a double expansion in $\sqrt{|\varepsilon|}$ and $N - N_{\rm c}$ give access to a different singly unstable FP which is expected to control the transition. Both methods give qualitatively similar pictures of the FRG flows: the critical and attractive FPs merge in some dimension $d^*_{\rm lc}(N) < d_{\rm lc}$ that is therefore the lower critical dimension of the QLRO-disordered transition. For the RF $O(2)$ model, both methods predict approximately the same lower critical dimension $d^*_{\rm lc} \approx 3.8(1)$, and thus, suggest that there is no Bragg glass phase in $d = 3$. However, one has to take caution when extrapolating results obtained for small $\sqrt{|\varepsilon|}$ and $N - N_{\rm c}$. Moreover, in contrast to the model of Refs. 26 and 20 which belongs to the so-called "hard-spin" models, the system studied in Ref. 22 corresponds to "soft spins", and thus, is expected to belong to a different universality class. In terms of vortices, the soft spin RF model allows for topological defects which destroy the Bragg glass.

Most studies of the RF and RA models are restricted to either short-range (SR) correlated disorder or uncorrelated pointlike defects. However, real systems often contain long-range (LR) correlated disorder or extended defects in the form of linear dislocations, planar grain boundaries, three-dimensional cavities, etc. Systems with anisotropic orientation of extended defects can be described by a model in which all defects are strongly correlated in ε_d dimensions and randomly distributed over the remaining $d - \varepsilon_d$ dimensions. The case $\varepsilon_d = 0$ is associated with uncorrelated pointlike defects, while extended columnar or planar defects are related to the cases $\varepsilon_d = 1$ and 2, respectively. The critical behavior of the $O(N)$ model with random-temperature-like extended defects was studied in Refs. 27,28,29,30 using a perturbative RG analysis in conjunction with a double expansion in $\varepsilon = 4 - d$ and ε_d.

In the case of an isotropic distribution of disorder, power-law correlations are the simplest example with the possibility for a scaling behavior with new FPs and new critical exponents. The critical behavior of systems with random-temperature disorder correlated as $1/x^{d-\sigma}$ for large separation x was studied in Refs. 31,32,33. The power law can be ascribed to extended defects of internal dimension σ, distributed with random orientation in d dimensional space. In general, one would probably not expect a pure power-law decay of correlations. However, if the correlations of defects arise from different sources with a broad distribution of characteristic length scales, one can expect that the resulting correlations will, over several decades, be approximated by an effective power law.[31] Power-law correlations with a noninteger value σ can be found in systems containing defects with fractal dimension σ.[34] For example, the behavior of $^4{\rm He}$ in aerogels is argued to be described by an XY model with LR correlated defects.[35] This is closely related to the behavior of nematic liquid crystals enclosed in a single pore of aerosil gel which was recently studied in Ref. 36, using the approximation in which the pore hull is considered a disconnected fractal. The FRG was used to investigate the statics and dynamics of elastic manifolds in media with LR correlated disorder in Ref. 37. The critical behavior of the $O(N)$ model with LR correlated RF was studied in Refs. 38,39,40. However, the methods used in these works fail to describe properly the case of SR correlated RF. Therefore, there is a necessity to reexamine the critical behavior of the LR RF model using methods which are successful in the SR case.

In the present paper, we study the LR correlated RF and RA $O(N)$ models using the FRG to one-loop order. The paper is organized as follows. In Sec. II, we introduce the LR RF and RA models, and derive the FRG equations. In Sec. III, we study the LR RF model. In Sec. IV, we consider the LR RA model and discuss the application to superfluid ^{3}He in aerogels. The final section summarizes our results.

II. MODEL AND FRG EQUATIONS

The large-scale behavior of the $O(N)$ symmetric spin systems at low temperatures can be described by the nonlinear σ model with the Hamiltonian

$$\mathcal{H}\left[\vec{s}\right] = \int d^d x \left[\frac{1}{2}(\nabla\vec{s})^2 + V(x,\vec{s})\right], \tag{1}$$

where $\vec{s}(x)$ is the N-component classical spin with a fixed-length constraint $\vec{s}^{\,2} = 1$. $\mathrm{V}(x,\vec{s})$ is the random disorder potential, which can be expanded in spin variables as follows:

$$V(x,\vec{s}) = \sum_{\mu=1}^{\infty}\sum_{i_1\ldots i_\mu} -h^{(\mu)}_{i_1\ldots i_\mu}(x) s_{i_1}(x)\ldots s_{i_\mu}(x). \tag{2}$$

The corresponding coefficients have simple physical interpretation: $h^{(1)}_i$ is a random field, $h^{(2)}_{ij}$ is a random second-rank anisotropy, and $h^{(\mu)}$ are general μth tensor anisotropies. As was shown in Ref. 14, even if the system has only finite number of nonzero $h^{(\mu)}$, the RG transformations generate an infinite set of high-rank anisotropies preserving the symmetry with respect to rotation $\vec{s} \to -\vec{s}$ if it is present in the bare model. For instance, starting with only second-rank anisotropy corresponding to the RA model, all even-rank anisotropies will be generated by the RG flow. We will reserve the notation RA for the systems which have this symmetry and the notation RF for the systems which do not. In the present work, we consider the case of Gaussian distributed long-range correlated disorder with zero mean and cumulants given by

$$\begin{aligned}\overline{h^{(\mu)}_{i_1\ldots i_\mu}(x) h^{(\nu)}_{i_1\ldots j_\nu}(x')} \; &= \; \delta^{\mu\nu}\delta_{i_1 j_1}\ldots\delta_{i_\mu j_\nu}[r^{(\mu)}_1\delta(x-x') \\ &\quad + r^{(\mu)}_2 g(x-x')],\end{aligned} \tag{3}$$

with $g(x-x') \sim 1/|x-x'|^{d-\sigma}$. For the sake of convenience, we fix the constant in Fourier space, taking $g(q) = 1/q^\sigma$.

To average over disorder, we introduce n replicas of the original system and compute the replicated Hamiltonian

$$\mathcal{H}_n = \int d^d x \Bigg[\frac{1}{2}\sum_a (\nabla \vec{s}_a)^2 - \frac{1}{2T}\sum_{a,b} R_1\big(\vec{s}_a(x)\cdot\vec{s}_b(x)\big) - \frac{1}{2T}\sum_{a,b}\int d^d x'\, g(x-x')\, R_2\big(\vec{s}_a(x)\cdot\vec{s}_b(x')\big)\Bigg], \quad (4)$$

where $R_i(z) = \sum_\mu r_i^{(\mu)} z^\mu$. The properties of the original disordered system (1) and (2) can be extracted in the limit $n \to 0$. According to the above definition of the RF and RA models, the functions $R_i(z)$ are arbitrary in the case of the RF and even for the RA. Power counting suggests that $d_{\rm lc} = 4+\sigma$ is the lower critical dimension for both models.[40]

At criticality or in the QLRO phase, the correlation functions of the order parameter exhibit scaling behavior. In contrast to the models with temperature-like disorder in the models under consideration, the connected and disconnected correlation functions may scale with different exponents. This reflects metastability and the breaking of the DR. For instance, the connected two-point function behaves as

$$\overline{\langle \vec{s}(q)\cdot\vec{s}(-q)\rangle - \langle\vec{s}(q)\rangle\cdot\langle\vec{s}(-q)\rangle} \sim q^{-2+\eta}, \quad (5)$$

while the disconnected function scales as

$$\overline{\langle \vec{s}(q)\rangle\cdot\langle\vec{s}(-q)\rangle} - \overline{\langle\vec{s}(q)\rangle}\cdot\overline{\langle\vec{s}(-q)\rangle} \sim q^{-4+\bar\eta}. \quad (6)$$

Here, $\vec{s}(q)$ is the Fourier component of the order parameter and the angle brackets stand for the thermal averaging. Schwartz and Soffer[41] proved that the exponents of the RF model obey the inequality $2\eta \geq \bar\eta$. Since in the RA case the coupling to disorder is bilinear, the Schwartz-Soffer inequality cannot be applied directly to η and $\bar\eta$. In the RA case, it is convenient to introduce the correlation functions of the form Eqs. (5) and (6) not for $\vec{s}$, but for the field $m_i = [s_i]^2 - 1/N$, and define exponents η_2 and $\bar\eta_2$ which satisfy the Schwartz-Soffer type inequality: $2\eta_2 \geq \bar\eta_2$.[26] Vojta and Schreiber[42] generalized this inequality to correlated RF and obtained a more restrictive bound $2\eta - \sigma \geq \bar\eta$ since $\sigma > 0$. For RA models similar arguments lead to $2\eta_2 - \sigma \geq \bar\eta_2$.

To derive the one-loop FRG equations, we straightforwardly generalize the methods developed in Refs. 14, 26 and 37 to model (4). We express the order parameter $\vec{s}_a$ as a combination[26]

$$\vec{s}_a(x) = \vec{n}_a(x)\sqrt{1-\vec{\pi}_a^{\,2}(x)} + \vec{\pi}_a(x) \quad (7)$$

of a fast field $\vec{\pi}_a$ fluctuating at small scales $\Lambda < q < \Lambda_0$ which is orthogonal to a slow field $\vec{n}_a$ of unit length, changing at scales $q < \Lambda$. Here, Λ_0 is the UV cutoff and $\Lambda \ll \Lambda_0$. The field $\vec{n}_a$ can be considered as the coarse-grained order parameter (local magnetization) whose fluctuations at low temperature are small, $\langle \vec{\pi}_a^2\rangle \ll 1$. Integrating out the fast variables $\vec{\pi}_a$, we rescale in such a way that the effective Hamiltonian of the slow fields $\vec{n}_a$ would have the structure of the bare Hamiltonian (4). It is convenient to change variable to $z = \cos\phi$. The FRG equations to first order in ε and σ are given by[43]

$$\begin{aligned}\partial_\ell R_1(\phi) &= -\varepsilon R_1(\phi) + \frac{1}{2}\big[R_1''(\phi)+R_2''(\phi)\big]^2 - AR_1''(\phi) \\ &\quad -(N-2)\Big\{2AR_1(\phi) + AR_1'(\phi)\cot\phi \\ &\quad -\frac{1}{2\sin^2\phi}\big[R_1'(\phi)+R_2'(\phi)\big]^2\Big\}, \end{aligned} \quad (8a)$$

$$\begin{aligned}\partial_\ell R_2(\phi) &= -(\varepsilon-\sigma)R_2(\phi) - \Big\{(N-2)\big[2R_2(\phi) \\ &\quad +R_2'(\phi)\cot\phi\big] + R_2''(\phi)\Big\}A, \end{aligned} \quad (8b)$$

where $\partial_\ell := -\partial/\partial\ln\Lambda$. We have absorbed the factor of $1/(8\pi^2)$ in redefinition of R and introduced

$$A = R_1''(0) + R_2''(0). \quad (9)$$

In terms of the variable ϕ, the functions $R_i(\phi)$ become periodic with period 2π in the RF case and π in the RA case. The flow equation for the temperature to one-loop order reads

$$\partial_\ell \ln T = -(d-2) - (N-2)A. \quad (10)$$

According to Eq. (10), the temperature is irrelevant for $d > 2$ and sufficiently small A. Although we expect $A = O(\varepsilon,\delta)$ in the vicinity of a FP, one has to take caution whether the found FP survives in three dimensions.[26] The scaling behavior of the system is controlled by a zero-temperature FP of Eqs. (8a) and (8b) $[R_1^*, R_2^*, A^*]$, such that $\partial_\ell R_i^* = 0$. An attractive FP describes a phase, while a singly (unidirectionally) unstable FP describes the critical behavior. The critical exponents are determined by the FRG flow in the vicinity of the FP and to one-loop order are given by

$$\eta = -A^*, \qquad \bar\eta = -\varepsilon - (N-1)A^*, \quad (11a)$$

$$\eta_2 = -(N+2)A^*, \quad \bar\eta_2 = -\varepsilon - 2NA^*. \quad (11b)$$

It is convenient to introduce the following reduced variables:

$$r_i(\phi) = R_i(\phi)/(\varepsilon-\sigma), \quad (12a)$$

$$a = A/(\varepsilon-\sigma), \quad (12b)$$

$$\hat\varepsilon = \varepsilon/(\varepsilon-\sigma). \quad (12c)$$

To check the stability of the FP $[r_1^*, r_2^*, a]$, we linearize the flow equations around this FP: $r_i(\phi) = r_i^*(\phi) + y_i(\phi)$ and obtain

$$\begin{aligned}\lambda y_1(\phi) &= -\hat\varepsilon y_1(\phi) + \big[r_1^{*\prime\prime}(\phi)+r_2^{*\prime\prime}(\phi)\big]\big[y_1''(\phi)+y_2''(\phi)\big] \\ &\quad -ay_1''(\phi) - a_0 r_1^{*\prime\prime}(\phi) - (N-2)\Big\{2a_0 r_1^*(\phi) \\ &\quad +a_0 r_1^{*\prime}(\phi)\cot\phi + 2ay_1(\phi) + ay_1'(\phi)\cot\phi \\ &\quad -\big[r_1^{*\prime}(\phi)+r_2^{*\prime}(\phi)\big] \\ &\quad \times\big[y_1'(\phi)+y_2'(\phi)\big]/\sin^2\phi\Big\}, \end{aligned} \quad (13a)$$

$$\begin{aligned}\lambda y_2(\phi) &= -y_2(\phi) - a_0\Big\{(N-2)\big[2r_2^*(\phi) + r_2^{*\prime}(\phi)\cot\phi\big] \\ &\quad +r_2^{*\prime\prime}(\phi)\Big\} - a\Big\{(N-2)\big[2y_2(\phi)+y_2'(\phi)\cot\phi\big] \\ &\quad +y_2''(\phi)\Big\}, \end{aligned} \quad (13b)$$

TABLE I: LR RF model above the lower critical dimension $d_{\rm lc} = 4+\sigma$. The FP values of $r_1^{*\prime\prime}(0)$, $r_1^{*\prime\prime}(\pi)$ and the relevant eigenvalue λ_1 computed numerically for different N and $\hat{\varepsilon}$. The last column is the relevant eigenvalue $\lambda_1^{\rm T}$ obtained from the truncated RG scheme of Ref. 39 and computed using Eq. (16).

N	$\hat{\varepsilon}$	$r_1^{*\prime\prime}(0)$	$r_1^{*\prime\prime}(\pi)$	λ_1	$\lambda_1^{\rm T}$
4	1.271	-1.0000	0.5811	1.271	2.429
	2	-0.4657	0.0891	1.218	2.000
	3	-0.3320	0.0041	1.198	1.618
	4	-0.2668	-0.0207	1.192	1.414
5	2	-0.1743	0.0192	1.167	1.366
	3	-0.1132	-0.0073	1.160	1.225
	4	-0.0819	-0.0138	1.144	1.158
6	2	-0.0941	0.0080	1.145	1.215
	3	-0.0549	-0.0058	1.127	1.135

where we have introduced the eigenvalues λ (measured in units of $\varepsilon-\sigma$) and defined $a_0 = y_1''(0)+y_2''(0)$. The FP is attractive if all λ_i fulfill the inequality $(\varepsilon-\sigma)\lambda_i < 0$. A singly unstable FP has only one eigenvalue λ_1 such that $(\varepsilon-\sigma)\lambda_1 > 0$, which determines the third independent exponent

$$\nu = 1/[\lambda_1(\varepsilon-\sigma)]. \tag{14}$$

This exponent characterizes the divergence of the correlation length in the vicinity of transition.

III. LONG-RANGE RANDOM FIELD $O(N)$ MODEL

We now focus on the phase diagram and critical behavior of the LR RF $O(N)$ model. Kardar *et al.*[38] studied the critical behavior of the LR RF $O(N)$ model using a $\bar{\varepsilon} = d_{\rm uc} - d$ expansion around the upper critical dimension $d_{\rm uc} = 6+\sigma$. They found that, to lowest order in $\bar{\varepsilon}$, the critical properties are that of a pure system in $d-2-\sigma$ dimensions; however, this generalized DR was found to fail at higher orders. Chang and Abrahams[39] applied to the LR RF $O(N)$ model a low-temperature version of the RG. Expanding around the lower critical dimension $d_{\rm lc} = 4+\sigma$, they obtained the RG recursion relations for the three parameters T, Δ_1, and Δ_2, which in our notation correspond to $\Delta_i = -R_i''(0)$. As we have shown in the previous section, the truncated RG neglects an infinite number of relevant operators. They found the nontrivial zero-temperature FP, which in our notation $\delta_i = \Delta_i/(\varepsilon-\sigma)$ reads

$$\delta_1^* = \frac{1}{(\hat{\varepsilon}-1)(N-3)^2}, \quad \delta_2^* = \frac{\hat{\varepsilon}(N-3)-N+2}{(\hat{\varepsilon}-1)(N-3)^2}. \tag{15}$$

The correlation length exponent ν is determined by Eq. (14), with the relevant eigenvalue λ_1 given by

$$\lambda_1^{\rm T} = \frac{N-2}{N-3} - \frac{\hat{\varepsilon}}{2} + \frac{\hat{\varepsilon}}{2}\sqrt{1+\frac{4[N-2-\hat{\varepsilon}(N-3)]}{\hat{\varepsilon}^{\,2}(N-3)^2}}. \tag{16}$$

Note that the expression for the relevant eigenvalue reported in Ref. 39 is incorrect and gives values which are several times

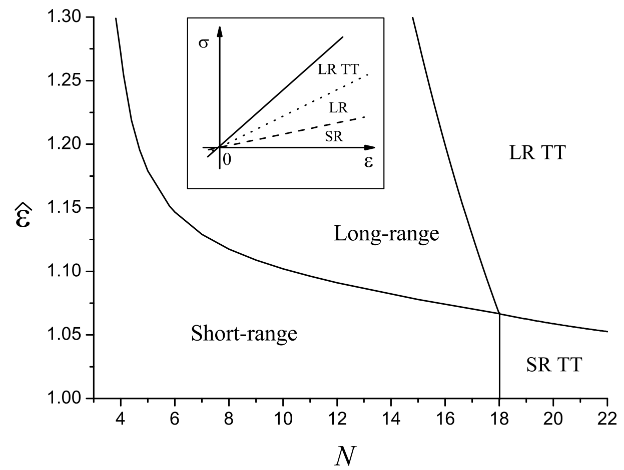

FIG. 1: The stability regions of various FPs corresponding to different patterns of the critical behavior above the lower critical dimension, $\varepsilon > \sigma$. The borderline between the SR and LR FPs is computed numerically using Eq. (22); the borderline between the SR TT and LR TT regions is given by Eq. (23) and that between the LR and LR TT regions by Eq. (25). Inset: Schematic phase diagram on the (ε, σ) plane for a particular value of $N \in (3, 18)$. The solid line is $\sigma = \varepsilon$; the dashed and dotted lines are given by Eqs. (22) and (25), respectively.

larger than that computed using Eq. (16). Although the truncated RG scheme when applied to the model with only SR correlated RF ($\Delta_2 = 0$) results in the DR FP $\Delta_{1\rm DR}^* = \varepsilon/(N-2)$ and exponent $\nu_{\rm DR} = 1/\varepsilon$, the exponent (16) differs from the generalized DR prediction $\nu_{\rm DR} = 1/(\varepsilon-\sigma)$. Nevertheless, one may doubt about the applicability of the truncated RG to the LR RF $O(N)$ model even if the DR is broken.

We now reexamine the long-distance behavior of the LR RF $O(N)$ model by means of the full one-loop FRG derived in Sec. II. Equation (8b) is linear in the function $r_2(\phi)$ and can be solved analytically. The FP solution fulfilling the RF boundary condition, i.e., 2π periodicity, is given by

$$r_2^*(\phi) = -r_2^{*\prime\prime}(0)\cos\phi, \tag{17}$$

which is an analytic function. Note that the analyticity of the LR part of the disorder correlator was also revealed in the elastic manifold problem.[37] This also gives us the FP value of a,

$$a_{\rm LRRF}^* = -\frac{1}{N-3}, \tag{18}$$

which, following Eqs. (11a), completely fixes the values of the critical exponents η and $\bar{\eta}$,

$$\eta_{\rm LR} = \frac{\varepsilon-\sigma}{N-3}, \quad \bar{\eta}_{\rm LR} = \frac{2\varepsilon-(N-1)\sigma}{N-3}. \tag{19}$$

Exponents (19) satisfy the generalized Schwartz-Soffer inequality at equality. This is at variance with the SR RF models, where the Schwartz-Soffer inequality was found to be strict,[19] but in agreement with the results for the LR RF spherical model with long-range interactions.[42] One may conjecture

that the generalized Schwartz-Soffer inequality is satisfied as equality for any N including the LR RF Ising model,[42] but there is no argument that this persists in higher orders of loop expansion.

Let us first discuss the critical behavior of the LR RF model above the lower critical dimension, $\varepsilon > \sigma$. Note that $-a^*_{\rm LRRF}$ exactly coincides with $\delta^*_1 + \delta^*_2$ given by Eqs. (15). Therefore the truncated RG would give the same values of η and $\bar{\eta}$ as the full FRG, at least to first order in ε and σ, if these exponents would be computed in Ref. 39. To obtain the function $r^*_1(\phi)$ and the amplitude $r^{*\prime\prime}_2(0)$, we integrate Eq. (8a) numerically. Since the coefficients of Eq. (8a) are singular at $\phi = 0$ and π, to compute the solution in the vicinity of these points, we use expansions of $r^*_1(\phi)$ in powers of $|\phi|$ and $(\pi-\phi)^2$, respectively. The expansion around 0 is completely determined by the value of $r^{*\prime\prime}_1(0)$,

$$\begin{aligned} r^*_1(\phi) &= \frac{2(N^2-4N+3)r^{*\prime\prime}_1(0)+N-1}{2(N-3)[(N-3)\hat{\varepsilon}-2(N-2)]} + \frac{r^{*\prime\prime}_1(0)\phi^2}{2} \\ &\pm \frac{\sqrt{r^{*\prime\prime}_1(0)(\hat{\varepsilon}-1)(N-3)^2+1}}{3(N-3)\sqrt{N+2}}|\phi|^3 + O(\phi^4), \end{aligned} \tag{20}$$

while to get the explicit expansion around π we need know $r^{*\prime\prime}_1(0)$ and $r^{*\prime\prime}_1(\pi)$:

$$\begin{aligned} r^*_1(\phi) &= (N-1) \\ &\times \frac{\left\{\left[r^{*\prime\prime}_1(0)+r^{*\prime\prime}_1(\pi)\right](N-3)+1\right\}^2 + 2r^{*\prime\prime}_1(\pi)(N-3)}{2(N-3)[(N-3)\hat{\varepsilon}-2(N-2)]} \\ &+ \frac{r^{*\prime\prime}_1(\pi)(\pi-\phi)^2}{2} + O[(\pi-\phi)^4]. \end{aligned} \tag{21}$$

From Eq. (20), we see that the SR part of the disorder correlator is nonanalytic at small ϕ, so that we have to distinguish the left and right derivatives. In what follows, we adopt $r^{(n)}_1(0) \equiv r^{(n)}_1(0^+)$. We use numerical integration to continue the solutions given by expansions (20) and (21) inside the interval $[0,\pi]$ and match them by adjusting the shooting parameters $r^{*\prime\prime}_1(0)$ and $r^{*\prime\prime}_1(\pi)$. Only the series with "+" in Eq. (20) can be matched with the solution computed using expansion (21). Following the third term of Eq. (20), the FP solution exists only if $-r^{*\prime\prime}_1(0) \leq \delta^*_1$. We found that this inequality is always strict, however, the difference $r^{*\prime\prime}_1(0)-(-\delta^*_1)$ becomes smaller in the limit of large N. Thus the truncated RG fails to give the correct values of $r^{*\prime\prime}_1(0)$ and $r^{*\prime\prime}_2(0)$ although it gives the correct sum. Consequently, the relevant eigenvalue λ_1 and the exponent ν obtained from the truncated RG are also expected to be incorrect. The computed values of $r^{*\prime\prime}_1(0)$ and $r^{*\prime\prime}_1(\pi)$ are shown in Table I.

Let us check the stability of FPs. First, we examine the stability of the SR FP $[r^*_{\rm SR}, r_2 = 0, a^*_{\rm SR} = r^{*\prime\prime}_{\rm SR}(0)]$ found numerically in Ref. 26. Linearized about the FP, Eq. (13b) can be solved analytically giving $y_2 = \cos\phi$ and $\lambda = -1 - a^*_{\rm SR}(N-3)$. The SR FP is stable against the introduction of LR disorder if $(\varepsilon-\sigma)\lambda < 0$. Taking into account that $\eta_{\rm SR} = -(\varepsilon-\sigma)a^*_{\rm SR}$ and $\bar{\eta}_{\rm SR} = -\varepsilon-(N-1)(\varepsilon-\sigma)a^*_{\rm SR}$, we can rewrite the criterion of the SR FP stability in the following form

$$\sigma < 2\eta_{\rm SR} - \bar{\eta}_{\rm SR}, \tag{22}$$

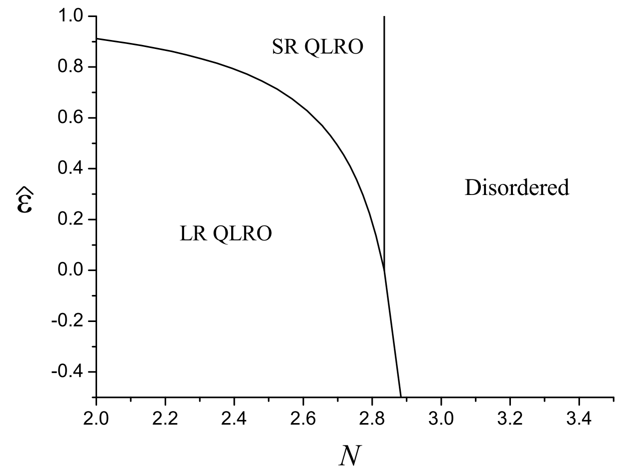

FIG. 2: Phase diagram of the RF model below the lower critical dimension, $d < 4+\sigma$ ($\varepsilon < \sigma$).

which was derived using general RG scaling arguments in Ref. 40. The regions of stability of the LR and SR FPs are shown in Fig. 1. Tissier and Tarjus (TT) argued that for $N > 18$, the cuspy SR FP becomes more than once unstable, but a different critical (singly unstable) SR TT FP arises with the function $R^*_1(\phi)$ being only $p \sim N$ times differentiable at the origin.[21] Although this weak nonanalyticity should reflect some metastability in the system,[20] it leads to the DR critical exponents. According to Eq. (22), the SR TT FP is stable with respect to the LR correlated disorder for $N > 18$ and

$$1 < \hat{\varepsilon} < (N-2)/(N-3). \tag{23}$$

We now check the stability of the LR RF FP $[r^*_1(\phi),\ r^*_2(\phi),\ a = -1/(N-3)]$. Substituting the LR FP in Eqs. (13a) and (13b), we obtain $y_2(\phi) = \cos\phi$ and

$$a_0 = -\frac{\lambda}{1 + r^{*\prime\prime}_1(0)(N-3)}. \tag{24}$$

To compute the eigenfunction $y_1(\phi)$ and eigenvalue λ, we solve Eq. (13a) numerically using shooting and imposing the 2π-periodic boundary condition. The obtained values of λ_1 are shown in Table I. As one can see from the table, the relevant eigenvalues λ_1 computed using the full FRG, and consequently, the exponent ν differ significantly from that obtained using the truncated RG. The critical exponents are expected to be continuous functions of ε, σ, and N. In the region controlled by the SR FP the correlation length exponent is $\nu_{\rm SR} = 1/\varepsilon$, independent on N.[21] The first line in Table I corresponds to a point on the borderline separating the regions of stability (see Fig. 1), and thus, on the borderline we indeed have $\nu_{\rm SR} = \nu_{\rm LR}$. We checked that the second eigenvalue λ_2 computed at the LR FP vanishes exactly on the borderline, indicating that the LR FP becomes twice unstable in the region dominated by the SR FP. We also looked for an analog of the TT phenomena in the presence of LR correlated disorder and found that for

$$\hat{\varepsilon} > \frac{2\sqrt{N+7}+6}{N-3} \tag{25}$$

there is a crossover to a new singly unstable LR FP of a TT type with the function $R_1^*(\phi)$ being only $p \sim N$ times differentiable at the origin and $R_2^*(\phi) \sim \cos\phi$. The values of $R_1''(0)$ and $R_2''(0)$ at the LR TT FP are simply given by Eq. (15). Thus, the LR TT FP leads to the critical exponents (19) and the relevant eigenvalue (16) obtained from the truncated RG.

Finally, we discuss the LR RF $O(N)$ model below the lower critical dimension, $\epsilon < \sigma$. For $N < 3$, the FRG equations have two attractive FPs: the SR and LR, which describe the SR QLRO and LR QLRO phases, respectively. The phase diagram computed from the stability analysis of different FPs is depicted in Fig. 2. The exponents characterizing the power-law behavior of the connected and disconnected correlation functions in the LR QLRO phase are given by Eq. (19). The physically interesting case $N = 2$ describing the Bragg glass phase in the presence of the LR correlated disorder was considered by one of the authors in Ref. 37. In the Bragg glass, the displacements $u(x)$ of a periodic structure (e.g., vortex lattice) grow logarithmically as $\overline{(u(x) - u(0))^2} = \mathcal{A}_d \ln|x|$. Note that in Ref. 37 the period was fixed to 1 while in the present work it is 2π, so that the relation between the universal amplitude $\mathcal{A}_d$ and the exponent η is given by $\eta = 2\pi^2\mathcal{A}_d$. For $N = 2$, the crossover from SR QLRO with the critical exponents $\eta_{\rm SR} = |\varepsilon|\pi^2/9$ and $\bar\eta_{\rm SR} = |\varepsilon|(1 + \pi^2/9)$ to LR QLRO with the exponents $\eta_{\rm LR} = |\varepsilon| + \sigma$ and $\bar\eta_{\rm LR} = 2|\varepsilon| + \sigma$ happens for $\hat\varepsilon < 9/\pi^2$ (see Fig. 2).

IV. LONG-RANGE RANDOM ANISOTROPY $O(N)$ MODEL

In this section, we study the LR RA case which corresponds to π-periodic functions $r_i(\phi)$. Equation (8b) has a family of (at most) π-periodic solutions which can be expressed as polynomials in $\cos\phi$ of power p with

$$a_p^* = \frac{1}{4p^2 + 2(p-1)(N-2)}, \qquad p = 1, 2, ... \quad (26)$$

For instance, for $p = 1$, we have

$$r_2^*(\phi) = \frac{1}{8N}[4r_1^{*\prime\prime}(0) - 1][N\cos^2\phi - 1], \quad (27)$$

$$a_{\rm LRRA}^* = \frac{1}{4}, \quad (28)$$

and for $p = 2$, we have $a_2^* = 1/(2N + 12)$ and

$$r_2^{(2)} = \frac{2(N+6)r_1^{(2)\prime\prime}(0) - 1}{8(N+1)(N+2)(N+6)} \times [3 - 6(N+2)\cos^2\phi + (N+2)(N+4)\cos^4\phi].$$

The stability analysis shows that due to the inequality $a_p^* < a_1^* \equiv a_{\rm LRRA}^*$ for $p \geq 2$ and $N \geq 2$, all FPs with $p > 2$ are unstable. It can be easily seen for $N = 2$ when the functions $r_2^{(p)}$ ($p \geq 2$) become (π/p)-periodic, and thus, are unstable with respect to a π-periodic perturbation. Henceforth we consider only the LR FP determined by Eqs. (27) and (28), which give the following values of the critical exponents

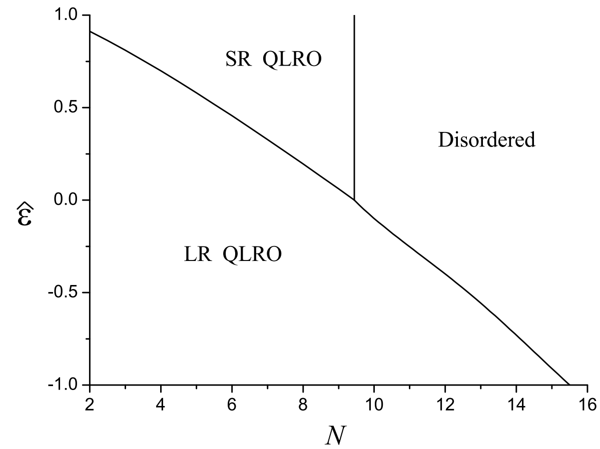

FIG. 3: Phase diagram of the RA model below the lower critical dimension, $d < 4 + \sigma$ ($\varepsilon < \sigma$).

$$\eta_{\rm LR} = \frac{\sigma - \varepsilon}{4}, \qquad \bar\eta_{\rm LR} = \frac{\sigma}{4}(N-1) - \frac{\varepsilon}{4}(N+3), \quad (29)$$

$$\eta_{2\rm LR} = \frac{(\sigma - \varepsilon)(N+2)}{4}, \quad \bar\eta_{2\rm LR} = -\varepsilon + \frac{N}{2}(\sigma - \varepsilon). \quad (30)$$

Exponents (30) satisfy the generalized Schwartz-Soffer inequality $2\eta_2 - \sigma \geq \bar\eta_2$ also at equality. Analogously to the LR RF model, in order to obtain the function $r_1^*(\phi)$, we solve Eq. (8a) numerically using shooting and imposing the π-periodic boundary condition. Since the coefficients of Eq. (8a) are singular at $\phi = 0$, we use an expansion of $r_1^*(\phi)$ in powers of $|\phi|$ which reads

$$r_1^*(\phi) = \frac{(N-1)[1 - 8r_1^{*\prime\prime}(0)]}{16(N - 2 + 2\hat\varepsilon)} + \frac{r_1^{*\prime\prime}(0)\phi^2}{2} \pm\sqrt{1 + \frac{16r_1^{*\prime\prime}(0)(\hat\varepsilon - 1)}{N+2}}\frac{|\phi|^3}{12} + O(\phi^4). \quad (31)$$

As one can see from Eq. (31), the SR disorder correlator $r_1^*(\phi)$ has a cusp at the origin. Only the solution with "+" in Eq. (31) fulfills all conditions.

We now check the stability of the SR RA FP with respect to the LR correlated disorder. Analogously to the RF case, linearized around the FP, Eq. (13b) allows for an analytical solution giving $y_2 = N\cos^2\phi - 1$ and $\lambda = -1 + 4a_{\rm SR}$. The SR FP is stable if $(\varepsilon - \sigma)\lambda < 0$, which can be rewritten as

$$\sigma < 2\eta_{2\rm SR} - \bar\eta_{2\rm SR}. \quad (32)$$

Above the lower critical dimension, $\varepsilon > \sigma$, inequality (32) holds for all $N > N_c = 9.4412$ so that the SR FP is stable with respect to the weak LR correlated disorder. Although the LR correlated disorder does not change the critical behavior for $N > N_c$, it can modify the critical behavior which should exist for $N < N_c$ but which is not accessible in the one-loop approximation.[19,20]

Below $d_{\rm lc}$, the LR RA model cannot develop a true long-range order, but there can exist different types of QLRO.

The SR QLRO phase is controlled by the SR RA FP, with $r_1^*(\phi) = r_{\rm SR}^*(\phi)$ and $r_2^*(\phi) = 0$ computed in Ref. 26. It was found to be stable in the subspace of the SR correlated disorder for $N < N_c = 9.4412$. We find that it is also stable with respect to the LR correlated disorder for σ fulfilling inequality (32). The new LR QLRO phase is controlled by an attractive LR RA FP with π-periodic functions $r_1^* \neq 0$ and $r_2^* \neq 0$, which we computed numerically by integrating Eq. (8a) with the initial condition given by Eq. (31) and using $r_1^{*\prime\prime}(0)$ as a shooting parameter. The exponents describing the correlation functions in the LR QLRO phase are given by Eqs. (29) and (30). Note that below $d_{\rm lc}$ we have $\eta > 0$. To check the stability of the LR RA FP, we substitute it in Eq. (13b) and obtain $y_2 = N\cos^2\phi - 1$ and

$$a_0 = \frac{2\lambda N}{4r_1^{*\prime\prime}(0) - 1}. \tag{33}$$

The eigenfunction $y_1(\phi)$ and eigenvalue λ are computed numerically using shooting. This allows us to determine the stability regions for different QLRO phases, which are shown in Fig. 3. As one can see from Fig. 3 in contrast to SR QLRO, LR QLRO can exist even for $4 < d < 4 + \sigma$ and $N > N_c$.

The above results may be relevant for the behavior of ^{3}He in aerogels. It has recently been observed in NMR experiments that the phase A of ^{3}He confined in aerogel exhibits two different types of magnetic behavior called c and f states.[44] Depending on the cooling, one can obtain either a nearly pure c state or a mixed $f + c$ state, which gives two overlapping lines c and f in the transverse NMR spectrum. Although the pure f state has not been observed, there is an evidence that the $f + c$ state is inhomogeneous and consists of regions with two different magnetic orders c and f.[44] The order parameter of the ^{3}He-A can be parametrized by the complex vector $\boldsymbol{\psi}$ and the real unit vector $\hat{\mathbf{d}}$, which characterize the orbital and magnetic anisotropy, respectively.[45] Only the orbital part of the order parameter given by the real unit vector $\hat{\mathbf{l}} = (i/2)[\boldsymbol{\psi}, \boldsymbol{\psi}^*]$ interacts with the aerogel matrix, which can be treated as quenched random anisotropy disorder. The spin part $\hat{\mathbf{d}}$ is not coupled directly to the disorder, but there is a weak spin-orbit (dipole) interaction between $\hat{\mathbf{l}}$ and $\hat{\mathbf{d}}$ which generates the NMR frequency shift. In Ref. 46, the existence of the two states was interpreted in terms of different "random textures" of the field $\hat{\mathbf{l}}$ in the Larkin-Imry-Ma state (QLRO phase in our notation). The measured dependencies of the transverse NMR signal on the tipping angle can be explained if one assumes that in the c state the vectors $\hat{\mathbf{l}}$ and $\hat{\mathbf{d}}$ are almost uncorrelated, while in the f state they are partially locked. This is possible if in the c state the characteristic length of the texture, i.e., the Larkin length, is much smaller than the characteristic length of the dipole interaction, $L \ll \xi_D$, while in the f state they are of the same order.[46] However, the nature of different random textures exhibiting different characteristic length scales is not clear. Alternatively, one can try to interpret the f state as a network of topological defects pinned by the aerogel.[46] We argue that these two different states may be the SR and LR QLROs found above in the LR RA model. Indeed, aerogel is a porous medium formed by tangled silicon strands which exhibit a fractal mass distribution. NMR and small-angle X-ray scattering (SAXS) experiments give the mass-to-distance relation $M \sim x^{d_f}$, with the fractal dimensions d_f varying in the range of 1.4 – 2.4 depending on the fabrication process. This scaling holds up to the fractal correlation length, which can exceed 100 nm.[47] Thus, the effective RA disorder is expected to be long-range correlated with $\sigma = d_f$ up to the scale of the fractal correlation length or even more.[35] As a result, the SR QLRO phase is unstable to formation of islands with LR QLRO (see Fig. 3). To compute the Larkin length, one has to solve the flow equations (8a) and (8b) starting from a particular bare disorder correlator and looking for the scale at which the cusp is developing.[37] We can estimate the Larkin length using Flory-type arguments.[26] Assuming that J is an elastic constant and R is a strength of disorder, we obtain $L_{\rm SR} \sim (J/R)^{2/\varepsilon}$ and $L_{\rm LR} \sim (J/R)^{2/(\varepsilon-\sigma)}$ for the SR and LR disorders, respectively. For aerogel, we have $\varepsilon = 1$ and $\sigma \approx 1.4 \ldots 2.4$ so that the Larkin lengths in both phases may differ significantly. This can explain the experimentally observed coexistence of regions with different spin states. Further investigations, however, are clearly needed.

V. SUMMARY

In this work, we investigated the long distance properties of the $O(N)$ model with random fields and random anisotropies correlated as $1/x^{d-\sigma}$ for large separation x. We derived the functional renormalization group equations to one-loop order, which allow us to describe the scaling behavior of the models below and above the lower critical dimension $d_{\rm lc} = 4 + \sigma$. Using a double $\varepsilon = d - 4$ and σ expansion, we obtained the phase diagrams and computed the critical exponents to first order in ε and σ. For the LR RF model, we found that the truncated RG developed in Ref. 39 to study the critical behavior above the lower critical dimension is able to give the correct one-loop values of exponents η and $\bar{\eta}$, but not the phase diagram and the critical exponent ν except for the region controlled by the weakly nonanalytic LR TT FP. Thus, although the truncated RG overcomes the dimensional reduction, it fails to reproduce all properties which can be obtained using the functional renormalization group. We found a new LR QLRO phase existing in the LR RF model below the lower critical dimension for $N < 3$ and determined the regions of its stability in the (ε, σ, N) parameter space. We obtained that the weak LR correlated disorder does not change the critical behavior of the RA model above $d_{\rm lc}$ for $N > N_c = 9.4412$, but can create a new LR QLRO phase below $d_{\rm lc}$. The existence of two QLRO phases in LR RA systems may explain the two different states of ^{3}He-A in aerogel observed recently in NMR experiments.[44] However, many questions are still open. In particular, the paramagnetic-ferromagnetic transition should exist also for $N < N_c$, though it was not found in the one-loop approximation. Even for the SR correlated disorder, it is still unclear whether it remains perturbative.[19,20] It would also be interesting to find a connection with the replica symmetry breaking picture.[48]

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